What does continuous-time model identification have to offer ?

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Abstract: Direct identification of continuous-time models from sampled data is now mature. The developed methods have proven successful in many practical applications and are available as user-friendly and computationally efficient algorithms in the CAPTAIN and CONTSID toolboxes for Matlab^{TM}. Surprisingly many practitioners appear unaware that such methods not only exist but may be better suited to their modelling problems. This paper discusses and illustrates with the help of real-life data the advantages of these direct schemes to continuous-time model identification.

Keywords: continuous-time model, discrete-time data, real-life data, system identification

1. INTRODUCTION

System identification is concerned with the problem of building mathematical models of dynamical systems based on observed inputs and outputs. Although dynamical systems in the physical world are naturally described in the continuous-time (CT) domain, most system identification schemes have been based on discrete-time models without much concern for the merits of natural continuous-time model descriptions. The development of continuous-time model-based system identification techniques originated in the middle of the last century but was overshadowed by the extensive developments of discrete-time (DT) model-based system identification methods. This was mainly due to the 'go completely digital' trend that was spurred by parallel developments in digital computers.

The last decade has witnessed a resurgence of interest devoted to direct CT modelling from DT data. Without relying any longer on analogue computers, the present techniques exploit the power of the digital tools. The developed methods have proven successful in many practical applications and are also available in user-friendly toolboxes for Matlab^{TM}. Exhaustive reviews of direct estimation methods can be found in Young [1981], Garnier et al. [2003], Rao and Unbehauen [2006], H. Garnier and L. Wang [2008], Garnier et al. [2011]. These approaches have recently shown better performance than DT model identification approaches in a number of simulation and real examples that compare CT and DT methods of identification applied to the same data sets, revealing some of the advantages of the CT approach, see Rao and Garnier [2002], **?**], Young et al [2008], Young [2011]. There are indeed several advantages of describing real CT systems by CT models and in identifying CT models directly from sampled data.

The main ones are as follows:

- CT models provide good physical insights into the system properties;
- CT models preserve *a priori* knowledge;
- CT models are defined by a unique set of parameter values that are not dependent on the sampling period;
- Direct CT methods can handle non-uniformly sampled data;
- Direct CT methods can cope easily with stiff systems: *i.e.* systems with eigenvalues and associated time constants that have different orders of magnitude, so that they contain both slow and fast dynamic modes of behaviour;
- Direct CT methods can easily handle fast sampled data;
- Direct CT methods can estimate 'fractional' time-delay systems.

In the present paper, these advantages are discussed and illustrated with the help of real-life data derived from different physical systems.

Amongst the direct identification approaches for CT *input-output* models, the *Refined Instrumental Variable* (RIVC) method [see e.g. Young and Jakeman, 1979, Young et al, 2008] is, in the authors' experience, the most efficient and has been used to identify the physical systems considered in this paper.

2. MOTIVATIONS FOR IDENTIFYING CT MODELS DIRECTLY FROM SAMPLED DATA

In the examples discussed in the following sub-sections, the 'coefficient of determination' R_T^2 based on the model errors, is defined as,

$$R_T^2 = 1 - \frac{\sigma_\varepsilon^2}{\sigma_y^2} \quad (1)$$

where σ^2 is the variance of simulated model error and σ_y^2 is the variance of measured output. This measures how much of the measured output variance is explained by the variance of deterministic model output. The measure R_T is defined in a similar manner but with the variances replaced by the associated standard deviations

$$R_T = 1 - \frac{\sigma_\varepsilon}{\sigma_y} \quad (2)$$

It measures how much of the measured output standard deviation is explained by the standard deviation of deterministic model output.

2.1 *CT Models Provide Good Physical Insights into the System Properties*

Most physical phenomena are viewed in a CT setting and the models of physical systems obtained from the application of physical laws are naturally in a CT form, normally linear or nonlinear differential equations. DT models can also be used to describe such systems: for instance, to simulate the system, to design a digital controller or develop a forecasting system. However, in a number of practical applications, the direct use of CT models is preferable. This is most often the case when one is looking for a model that represents an underlying CT physical system, and wish to ascertain the values of parameters that have a physical meaning, such as time constants, reaction times, elasticity, mass, etc. While these parameters are directly linked to the CT model, the parameters of DT models are a function of the sampling interval and do not have the same direct physical interpretability.

Such physical interpretation is a major aspect of *Data-Based Mechanistic* (DBM) modelling [see e.g. Young, 1998, 2011]. A good example of a DBM model that is developed using CT identification is described in Price et al. [1999]. Here, the change in temperature of air flowing through a ventilated chamber is identified from the data shown in Fig. 1, using the RIVC algorithm, as implemented by the rivcbj routine in the CAPTAIN Toolbox [1] for Matlab$^{\text{TM}}$. This identifies a second order CT system with the following estimated TF model:

$$\Delta T(t) = \frac{1.9519s + 0.08828}{s^2 + 2.9487s + 0.11045} \Delta T_i(t) \quad (3)$$

where s is the derivative operator. This can be decomposed into a negative feedback connection of two first order processes with steady-state gains of $\{0.669,\ 0.130\}$ and widely spaced time constants of $\{0.343,\ 26.35\}$ minutes. The reason for the presence of the longer time constant in this model is that the installation is not effectively insulated by the outer chamber which, instead, acts as a temperature buffer zone with its own heat transfer characteristics.

Now, invoking the classical theory of heat transfer, it is straightforward to develop differential equations for the temperature changes in the main chamber and the buffer zone that convert to the following transfer functions (TFs):

[1] The fully functional CAPTAIN Toolbox can be downloaded from `http://captaintoolbox.co.uk/Captain_Toolbox.html`.

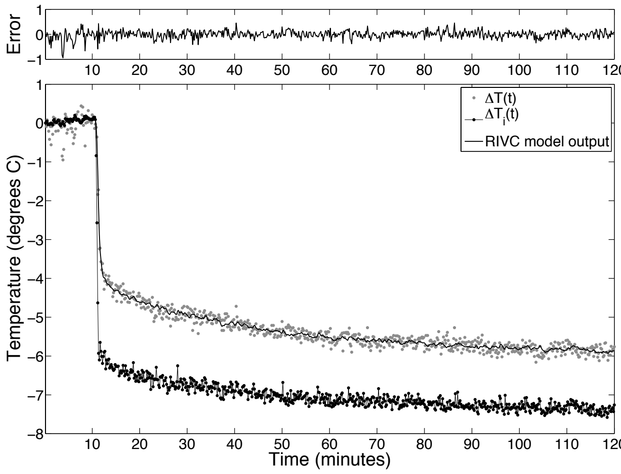

Fig. 1. Ventilation chamber experiment: input temperature $\Delta T_i(t)$; output temperature $\Delta T(t)$; and rivcbj estimated model output compared with $\Delta T(t)$.

$$\Delta T(t) = \frac{1}{(s + \alpha_1)}(\beta_1 \Delta T_i(t) + K_1 \Delta T_b(t))$$

$$\Delta T_b(t) = \frac{K_3}{(s + K_2 + K_3)} \Delta T(t)$$

where $\Delta T_b(t)$ are the changes of temperature in the buffer zone. Combining these two TFs, we obtain the following second order TF model for the complete chamber-buffer zone system:

$$\Delta T(t) = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2} \Delta T_i(t) \quad (4)$$

where the various parameters are functions of the classical heat transfer parameters (e.g. specific heat capacities, thermal conduction coefficients etc., as described in Price et al. [1999]). Clearly, this TF model is in exactly the same structural form as the estimated TF in (3) and so, in DBM modelling terms, it can be considered as one particular physical interpretation of the data-based model that has scientific credibility.

Note that the classical heat transfer coefficients are not uniquely identifiable from the relationships between the parameters in equations (3) and (4). On the other hand, the heat transfer dynamics of the chamber are completely specified by the DBM model parameters, which can be interpreted as specific combinations of these classical parameters. Consequently, this DBM model represents an alternative approach to modelling the system in physically meaningful, albeit not the normal, classical terms. And the model, in this efficiently parameterized, identifiable form, is entirely adequate for both understanding the heat transfer dynamics of the chamber and designing a ventilation control system. It provides, in other words, an alternative way of presenting heat transfer theory within the context of imperfectly mixed flow processes, such as those encountered in real forced ventilation systems.

2.2 *Direct CT Methods Can Cope with Non-uniformly Sampled Data*

In some situations, it is difficult to obtain equidistant sampled data. This sampling situation arises in medicine, environmental science, transport and traffic systems, astrophysics and other areas, where the instant of a measurement is not under control of the experimenter, or where uniform sampling is practically impossible. In these cases, the CT model, which is independent of a sampling period, is more flexible than a DT model. The standard DT linear model cannot represent the system, since it is inherently based on a constant sampling period. The CT model, however, represents the system at every time-instant. The measurements are basically just points on a continuous line, which do not need to be equidistantly spaced. The example described below is quite representative of irregularly sampled data situations that appear in system biology.

Figure 2 depicts the basic material used for *in vitro* experiments when studying the uptake kinetics of a photosensitising drug into living cancer cells. Cells are seeded in culture wells and are exposed at time $t_0 = 0$ to a photosensitising drug D. The purpose of this study is the estimation of the uptake yield (ρ) and the initial uptake rate (v_0). These biological parameters allow the biologists to discriminate the uptake characteristics of different photosensitisers and thus to choose the suitable photosensitising agent for the treatment of a given cancer cell line.

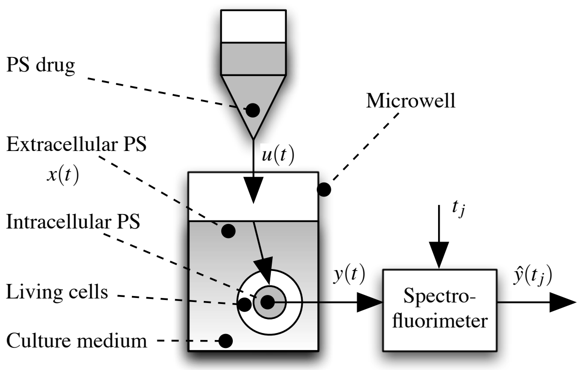

Fig. 2. Biological process.

The input $u(t)$ is a step that corresponds to the amount of drug injected into the well from time t_0. The magnitude of the step is given by $u_0 = 5 \times 10^{-3} \mu\text{mol} \cdot \text{L}^{-1}$. $x(t)$ and $y(t)$ denote the extracellular quantity of photosensitising drug and the amount of drug absorbed by the cells, respectively. The process output is $y_m(t)$, the measurement of $y(t)$, given by a spectrofluorimeter at times $\{t_k\} = \{1, 2, 4, 6, 8, 14, 18, 24h\}$. Therefore, we are confronted with a model identification problem from non-uniformly sampled data. Two experiments have been carried out with two different protein concentrations $[Se] = 0\%$ et $[Se] = 9\%$ in the culture medium.

The *in vitro* uptake of the photosensitising agent into cancer cells can be described by simple linear first-order model. The model parameters can be directly estimated by using the RIVC algorithm, as implemented by the srivc routine in the CONTSID Toolbox [2] for Matlab$^{\text{TM}}$ on two *in vitro* irregularly data sets. The estimated transfer function models for the two protein concentrations take the form

$$\hat{G}_{0\%}(s) = \frac{162.7(\pm 21.45)}{s + 0.333(\pm 0.056)} \qquad (5)$$

$$\hat{G}_{9\%}(s) = \frac{41.1(\pm 7.18)}{s + 0.535(\pm 0.12)} \qquad (6)$$

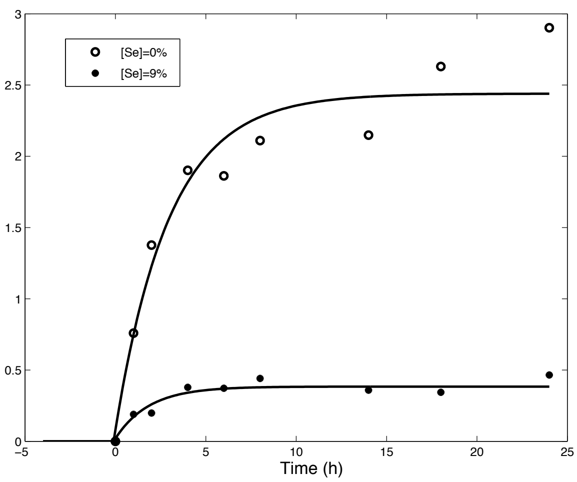

Fig. 3. Irregularly measured ('•' and '∘') and simulated uptake model responses (solid lines)

Figure 3 shows the good agreement between the two data sets and the simulated uptake model responses obtained from $\hat{G}_{0\%}(s)$ and $\hat{G}_{9\%}(s)$.

2.3 Direct CT Methods Can Cope Easily with 'Stiff' Systems

The combination of slow and fast dynamic modes of behaviour in stiff systems makes the selection of the data sampling interval rather difficult: on the one hand it seems to require rapid sampling, with a small sampling interval, in order to capture the fast dynamics; but, on the other hand and particularly in the case of DT models, it appears to require sampling slowly, with a long sampling interval, in order to identify the slow dynamic modes. Of course, these requirements are incompatible, so it is necessary to select a compromise of some sort: normally, the selection of a relatively short sampling interval. Unfortunately, in the case of DT models, this can often mean that one eigenvalue of the identified TF model may be very close to the unit circle in the complex z-plane. And this can deleteriously affect the quality of the model, as we see in the example below and in the next sub-section 2.4.

In contrast to DT estimation, direct CT model identification algorithms work well with rapidly sampled data from stiff systems. A case in question is the ventilation chamber system considered in the previous sub-section 2.1, which happens to be a stiff system, with identified time constants of 0.343 and 26.35 minutes.

Figure 4 again compares the output of the rivcbj estimated model with the measured output temperature but also shows the outputs of models estimated by two DT estimation algorithms: the rivbj algorithm in the CAPTAIN Toolbox, and the PEM algorithm [Ljung, 1999], as implemented by the pem routine in the Matlab$^{\text{TM}}$ SID Toolbox.

[2] The fully functional CONTSID Toolbox can be downloaded from `http://www.cran.uhp-nancy.fr/contsid/`.

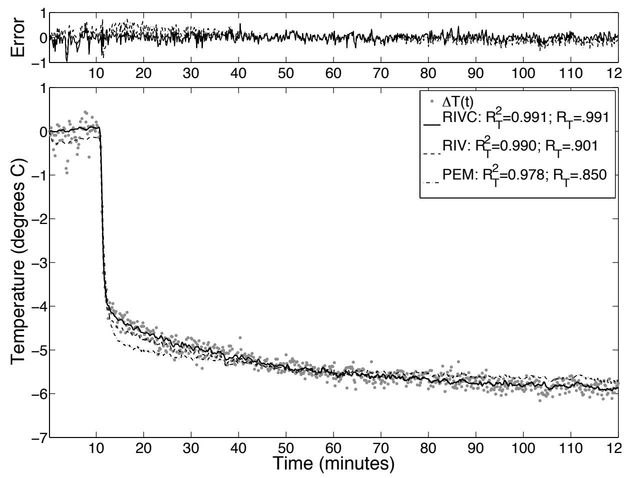

Fig. 4. Comparison of the measured and estimated model temperatures at the chamber outlet.

The rivbj estimated DT model's explanation of the data, with $R_T^2 = 0.9902$ (see eq. (1)) is poorer than that of the rivcbj estimated model, and this is particularly emphasised by its R_T measure (see eq. (2)) of 0.9010 (i.e. only 90,1% of the output standard deviation is explained by the model output), which compares with $R_T = 0.991$ in the rivcbj case.

In the case of the pem results, the explanation of the data is visibly quite poor, as reflected in a R_T^2 value of 0.978 and a R_T of 0.85. It has been noted previously that pem can perform poorly when applied to data from stiff systems [see e.g. Young, 2008, 2011]: it appears to be caused by multiple minima and initiation problems [Rao and Garnier, 2002, Ljung, 2003]. This is demonstrated if pem is initiated with the model obtained when the rivcbj estimated CT model is converted to DT using the c2d routine in the Matlab^{TM} SID Toolbox: this initialises the pem search routine very close to the optimum and yields a DT model with $R_T^2 = 0.9905$ and $R_T = 0.9023$, which is now similar to the rivbj estimated model. The most important aspect of these results, however, is that both DT algorithms do not perform as well as the CT algorithm, illustrating the advantages on CT estimation when applied to stiff system data, as in this example.

2.4 Direct CT Methods Can Cope Easily with Fast Sampled Data

An early discussion on the selection of the sampling interval ΔT for DT model identification is given in Aström [1969], which shows that there is a safe band of sampling intervals at sampling intervals that are well removed from the critical Nyquist-Shannon 'upper limit' and that will ensure low variance parameter estimates. There are several 'rules-of thumb' that assist in the selection of the sampling interval within this band: for instance, that the sampling frequency should be ten times the bandwidth [Ljung, 1999]. In the case of CT models, however, the selection is much more straightforward since there is effectively no lower limit on ΔT, except in relation to the size of the resultant data set.

In order to consider the effects of the sampling interval on the estimation of CT model parameters, consider the following model, which is based originally on hourly rainfall and flow data from the River Wye at Cefn Brwyn (see Littlewood et al, 2010). The rivcbj estimated CT model in the decomposed form is as follows:

$$y(t) = \left[\frac{0.162}{s + 0.3426} + \frac{0.00263}{s + 0.00553} \right] u(t) + \xi(t) \qquad (7)$$

which is, once again, a stiff system, with a quick time constant of $T_q = 2.92$ hours and a slow time constant of $T_s = 180.8$ hours. The bandwidth of the quick-flow mode, which dictates the sampling strategy in the present context, is 0.3426 rad/hour or $0.3426/2\pi$ cycles/hour. Consequently, for this mode, $1/\Omega = 2\pi/0.3426$ hours, where Ω is the Nyquist frequency, and the critical sampling interval associated with Ω is $\pi/0.3426 = 9.17$ hours the slow-flow mode.

The model (7) is simulated in Matlab^{TM}, using the lsim routine at a very small and, therefore, accurate sampling interval of one second, over a total simulation time of 6500 hours (23,400,000 seconds!). In order to ensure good parametric identifiability, the input is selected as a 'persistently exciting', repeated unit step signal with six steps over the 6500 hour total period so that, at each step (which excites all frequencies), the full dynamic behaviour is simulated, with the modelled flow output reaching near steady state each time and so providing good estimation of the steady state gain.

In order to simulate uncertainty in the measurements, the additive noise signal, $\xi(t)$ is selected as a zero mean, serially uncorrelated random variable (discrete-time white noise sampled at one second) with a noise to signal ratio by standard deviation of 0.1. In this way, the complicating effects of colour in the noise are avoided and the estimation results reflect more clearly the effects of the sampling interval. This means that the results obtained here are probably the best that could be obtained and that a less persistently exciting signal and a more coloured noise sequence would probably lead to less estimation accuracy than is seen in the results presented below.

In order to investigate the effects of different sampling interval values, the one second sampling interval data generated by the above simulation experiment is sub-sampled with sub-sampling intervals ranging from 5 seconds to 10 hours. For each sub-sampled data set obtained in the above manner, the continuous-time model parameters are estimated in two ways. First, directly, using the rivcbj algorithm in the CAPTAIN Toolbox; second, indirectly by discrete-time model identification, using the rivbj algorithm in CAPTAIN and the pem algorithm in the SID Toolbox, to obtain estimates of the parameters in the DT model (with structure [2 2 1]); and then using the d2c routine in Matlab^{TM} to convert this model to continuous-time with the zero-order hold (ZOH) assumption.

Figure 5 shows the estimates of the two time constants T_q and T_s plotted against the sampling interval. The true values of the time constants are shown as the dash-dot lines and, as expected, the direct rivcbj estimate is very much superior for the small sampling intervals (smaller than 10 here), with rivbj and pem identification producing very erratic results at these very small sampling intervals where the estimated DT model eigenvalues (roots of the denominator polynomial) are very close to the unit circle in the complex z domain (so that the model increasingly appears like an integrator).

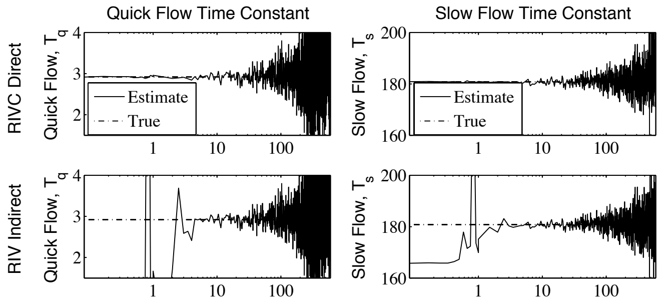

Fig. 5. Direct RIVC and indirect RIV estimates of the quick T_q and slow T_s time constants against the sampling interval.

At the other extreme, all the estimation algorithms appear to perform similarly, with increasing variance as the critical Nyquist-Shannon frequency is approached. However, the plot does not reveal that the rivcbj results do exhibit some failures as the sampling interval becomes too large to allow for reliable CT identification. Of course, the increased variance with increasing sampling interval is partly due to the fact that the sample size is reducing as the sub-sampling interval is increased: the original sample size of 23,400,000s is reduced to only 650 samples when the sub-sampling interval is ten hours.

The conclusion from these results is that the direct CT model identification is more reliable, over a wider range of sub-sampling intervals, than indirect estimation using DT methods.

2.5 Direct CT Methods Include Inherent Data Filtering

The importance of explicitly prefiltering data in DT transfer function model identification algorithms that exploit a 'pseudo-linear regression' approach, such as the rivbj in CAPTAIN and the iv4 algorithm in SID, was discussed by Young [1976] and Söderström and Stoica [1983]. They showed that it can improve the statistical efficiency of the parameter estimates and, if used judiciously, can result in optimal identification algorithms. The reason is fairly obvious since the prefilters, when iteratively updated, selectively attenuate noise outside the bandpass of the system and pre-whiten noise that remains present within this bandpass [see e.g. Young, 2011].

While an explicit prefiltering strategy is inherent in the RIV method of DT model identification, it is not the norm in other DT methods, such as the PEM algorithm (although such prefiltering does occur in the derivation of the gradient measures used in the gradient-based optimization). Consequently the user is then confronted by a choice of whether or not to add explicit prefiltering of some kind [Ljung, 2003]. This can be contrasted with the situation in CT identification, where the prefiltering is essential and has two combined roles: in addition to its original 'state-variable filtering' role [Young and Jakeman, 1979, H. Garnier and L. Wang, 2008] for reconstructing the filtered time-derivatives within the bandwidth of the system to be identified, it became clear during the early development of the RIVC algorithm that it could be exploited to perform the same, statistically meaningful prefiltering role in CT identification, leading to the development of the SRIVC and full RIVC algorithms (see previous references).

2.6 Direct CT Methods Can Estimate 'Fractional' Time-delay Systems

Time delays that are less than the time unit of the CT model can be accommodated in two ways. First and most simply, if the sampling interval is small and less than the time unit, as we have seen it can be in the case of CT model identification, then quite accurate fractional time delays can be identified as an integral number of these small sampling intervals. For instance, in the case of the ventilation chamber experiment in sub-sections 2.1 and 2.3, the time unit is minutes and the sampling interval is 10 seconds or 1/6 of a minute; while in the rainfall-flow model of sub-section 2.4, the normal time unit is hours and sampling intervals can be less than this.

If the CT model identification is implemented to include fractional time delays directly in the simulated model, then the value of this time delay can be estimated as another parameter. The presently available general CT identification algorithms, such as rivcbj and srivc, do not allow for this, although it is a possibility for future implementations. However, special purpose CT model identification algorithms, including fractional time delays, have been developed: see e.g. Young and Garnier [2006], where the possibility of a fractional time delay in a CT model of the relationship between CO_2 emissions (arising from both the use of carbon fuels and land-use changes) and the perturbations in atmospheric carbon dioxide partial pressure, pCO_2, is identified using the leastsq optimization tool in Matlab$^{\text{TM}}$, with the CT model simulated to include a variable time delay, using the linsim routine and a Simulink$^{\text{TM}}$ model.

2.7 CT Models Preserve a priori Knowledge

The *a priori* knowledge of the relative degree is easy to accommodate in CT models and, therefore, allows the identification of more parsimonious models than in discrete-time. For example, consider a second order CT transfer function

$$G(s) = \frac{1}{ms^2 + bs + k} \tag{8}$$

where the parameters represent mass, elasticity and friction. The DT model of the same process takes the form

$$G(z) = \frac{b_0 z + b_1}{a_0 z^2 + a_1 z + a_2}. \tag{9}$$

where z denotes the forward shift operator. Additional parameters are introduced (which need to be estimated) in the numerator of the DT transfer function by the sampling process. This simple example illustrates that it is not easy to preserve *a priori* knowledge about the model structure in DT model identification and the identification of CT models is definitely advantageous in this regard.

2.8 Discretisation may Render CT Models Non-minimum Phase and the Intersample Behaviour Assumption for the Input may be Critical for DT models

The question of parameter transformation between a DT description and a CT representation is non-trivial. First, the zeros of the DT model are not as easily translatable to CT equivalents as the poles [Åström et al., 1984]; second, due to the discrete nature of the measurements, they do not contain all the information about the CT signals. In order to describe the signals *between* the sampling instants, some additional assumptions have to be made: for example, assuming that the excitation signal is constant within the sampling intervals (the zero-order hold assumption). However, violation of these assumptions may lead to severe estimation errors. The reader is invited to run the demonstration program available in the CONTSID toolbox where some of the advantages discussed here are illustrated.

3. CONCLUSION

This paper has discussed and illustrated the many advantages of using CT models and related direct CT model identification methods. First and most importantly, it provides differential equation models that conform with models used in most scientific research, where conservation equations are normally formulated in terms of differential equations. It is also a model defined by a unique set of parameter values that often have physically meaningful units and are not dependent on the sampling interval, so eliminating the need for conversion from discrete to continuous time that is an essential element of indirect approaches to estimation based on discrete-time model estimation. These direct continuous-time model identification methods have also been developed to handle mild non-uniformly sampled data, dominant system modes with widely different natural frequencies (stiff systems), rapidly sampled data, or when the input does not respect the zero-order hold assumption. Last, but not least, these methods have proven successful in many practical applications and are available as computationally efficient algorithms in the CONTSID and CAPTAIN toolboxes for Matlab$^{\mathrm{TM}}$.

REFERENCES

K. J. Aström. On the choice of sampling rates in parametric identification of time series. *Information Science*, 1 (1):273–278, 1969.

K.J. Åström, P. Hagander, and J. Sternby. Zeros of sampled systems. *Automatica*, 20(1):31–38, 1984.

H. Garnier, M. Mensler, and A. Richard. Continuous-time model identification from sampled data. Implementation issues and performance evaluation. *International Journal of Control*, 76(13):1337–1357, 2003.

H. Garnier and L. Wang (Eds.). *Identification of Continuous-Time Models from Sampled Data*. Springer-Verlag, London, 2008.

H. Garnier, T. Söderström, and J. Yuz. Special issue on "Continuous-time model identification". *IET Control Theory & Applications*, 5(7), May 2011.

I. G. Littlewood, P. C. Young, and B. F. W. Croke. Preliminary comparison of two methods for identifying rainfall–streamflow model parameters insensitive to data time-step: the Wye at Cefn Brwyn, Plynlimon, Wales. In Claire Walsh (Ed.), *Role of Hydrology in Managing Consequences of a Changing Global Environment*, pages 539–543, Newcastle, U.K., 2010.

L. Ljung. *System Identification. Theory for the User.* Prentice Hall, Upper Saddle River, 2nd edition, 1999.

L. Ljung. Initialisation aspects for subspace and output-error identification methods. In *European Control Conference (ECC'2003)*, Cambridge, U.K., 2003.

L. Price, P.C. Young, D. Berckmans, K. Janssens, and J. Taylor. Data-based mechanistic modelling and control of mass and energy transfer in agricultural buildings. *Annual Reviews in Control*, 23:71–82, 1999.

G.P. Rao and H. Garnier. Numerical illustrations of the relevance of direct continuous-time model identification. In *15th Triennial IFAC World Congress on Automatic Control*, Barcelona (Spain), July 2002.

G.P. Rao and H. Unbehauen. Identification of continuous-time systems. *IET Control Theory & applications*, 153 (2):185–220, 2006.

T. Söderström and P. Stoica *Instrumental Variable Methods for System Identification.* Springer Verlag, New York, 1983.

P. Stoica. and T. Söderström Optimal instrumental variable estimation and approximate implementations. *IEEE Transactions on Automatic Control*, AC-28(7): 757–772, 1983.

P. C. Young. Some observations on instrumental variable methods of time-series analysis. *International Journal of Control*, 23:593–612, 1976.

P. C. Young and A. J. Jakeman. Refined instrumental variable methods of time-series analysis: Parts I, II and III. *International Journal of Control*, 29, 29, 31:1–30; 621–644; 741–764, 1979, 1980.

P. C. Young. Parameter estimation for continuous-time models - a survey. *Automatica*, 17(1):23–39, 1981.

P. C. Young. Data-based mechanistic modeling of environmental, ecological, economic and engineering systems. *Environmental Modelling and Software*, 13:105–122, 1998.

P. C. Young and H. Garnier. Identification and estimation of continuous-time, data-based mechanistic models for environmental systems. *Environmental Modelling & Software*, 21:1055–1072, 2006.

P. C. Young. The refined instrumental variable method: unified estimation of discrete and continuous-time transfer function models. *Journal Européen des Systèmes Automatisés*, 42:149–179, 2008.

P. C. Young, H. Garnier, and M. Gilson. Refined instrumental variable identification of continuous-time hybrid Box-Jenkins models. In H. Garnier and L. Wang (Eds.), *Identification of Continuous-Time Models from Sampled Data*, pages 91–131. Springer-Verlag: London, 2008.

P. C. Young. The estimation of continuous-time rainfall-flow models for flood risk management. In Claire Walshe (Ed.), *Role of Hydrology in Managing Consequences of a Changing Global Environment*, pages 303–310, Newcastle, U.K., 2010.

P. C. Young. *Recursive Estimation and Time-Series Analysis: an Introduction for Students and Practitioners.* Springer-Verlag, Berlin, 2011.

L. Wang and H. Garnier (Eds.) *System Identification, Environmetric Modelling and Control System Design.* Springer-Verlag, November 2011.