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# Recursive Estimation and Time-Series Analysis

An Introduction for the Student and  
Practitioner

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*To Wendy*



## Preface

This is a revised version of my 1984 book of the same name but, because so much time has elapsed since the publication of the first version, it has been considerably modified and enlarged to accommodate all the developments in recursive estimation and time series analysis that have occurred over the last quarter century. Also over this time, the CAPTAIN Toolbox for recursive estimation and time series analysis has been developed by my colleagues and I at Lancaster, for use in the Matlab<sup>TM</sup> software environment (see Appendix G). Consequently, the present version of the book is able to exploit the many computational routines that are contained in this widely available Toolbox, as well as some of the other routines in Matlab and its other toolboxes.

The book is an introductory one on the topic of recursive estimation and it demonstrates how this approach to estimation, in its various forms, can be an impressive aid to the modelling of stochastic, dynamic systems. It is intended for undergraduate or Masters students who wish to obtain a grounding in this subject; or for practitioners in industry who may have heard of topics dealt with in this book and, while they want to know more about them, may have been deterred by the rather esoteric nature of some books in this challenging area of study. As such, it can also be considered as a primer for the eventual reading of these more advanced theoretical texts on the subject. However it should be emphasized that the book also contains a considerable amount of novel material which does not appear in any other texts on the subject.

There are many people who have influenced my work over many years and who I wish take the opportunity to thank. First, my colleagues in the Environmental Science Department at Lancaster, Keith Beven, Arun Chotai, Wlodek Tych, Andrew Jarvis and Nick Chappell, as well as other colleagues in other Departments: Granville Tunnicliffe-Wilson, Peter Diggle and Jon Tawn in Mathematics and Statistics; James Taylor in Engineering; and Robert Fildes in the Management School. Of these, particular thanks are due to Keith Beven, who has continually encouraged my trespass into the hydrological world and has heavily influenced my research on both *Data-Based Mechanistic* (DBM) modelling and flood forecasting (see 12); Wlodek Tych, one of the major architects of the CAPTAIN Toolbox, who persuaded me to try Matlab out in the first place and who has solved many Matlab problems for me;

and Arun Chotai, who has been a good friend over many years and shared with me the systems and control teaching in the Department.

Whilst at Lancaster I have worked with research students and assistants, all of whom have enriched my life and thinking but are too numerous to mention. Suffice it to say that those who have particularly influenced my time series and modelling research are Cho Ng, Matthew Lees, Stuart Parkinson, Laura Price, Chris Fawcett, Miranda Foster, Paul McKenna, Andrew Jarvis, Renata Romanowcz and, more recently, Dave Leedal and Paul Smith. Particular thanks are due to James Taylor and Diego Pedregal who both worked with me for a number of years and have been of great assistance in the development of the CAPTAIN Toolbox.

Before I came to Lancaster I was a Professorial Fellow at the Australian National University (ANU), in Canberra. It was there that I met and discussed many things with Ted Hannan, one of the all-time greats of time series analysis; and, with him, supervised Vic Solo, whose significant contributions to recursive time series analysis and signal processing are well known. Also I worked with Tony Jakeman, now a long-time and trusted friend, on the development of the recursive *Refined Instrumental Variable* (RIV) approach to time series analysis that figures so strongly in this book (see chapters 6 to 10). I was also influenced by discussions with Mike Osborne, David (Dingle) Smith and Tom Beer, who helped develop the *Aggregated dead Zone* (ADZ) model (see chapters 6, 8 and 12). Being in Australia also allowed me to continue my friendship with Graham Goodwin at Newcastle, NSW, whose contributions to the theory and practice of time series analysis and automatic control have been so impressive for the past forty years, and whose work has influenced me to a great extent. I were also able to continue my friendship, first established in California during the 1960s, with Neville Rees, with whom I have had so many useful discussions on adaptive control and, latterly with Chris Lu, on large simulation model emulation and control (see chapter 12).

I moved to the ANU from Cambridge, where I was a Lecturer in Engineering and Fellow of Clare Hall. It was here that I consolidated my earlier Ph.D research at Cambridge on recursive estimation and self-adaptive control. This Ph.d research was continually encouraged by John Coales, a great friend who was so influential in my career; and Howard Rosenbrock, who excited my interest in Karl Friedrich Gauss (who first developed the recursive least squares algorithm: see Appendix A). Over the six years that I was at Cambridge, I supervised numerous research students who helped to shape my later research interests, such as Karl Neethling, Joseph Kittler, John Naughton and Paul Whitehead. But no one had a more profound influence on my future career than Bruce Beck, whose enthusiasm for pursuing research on environmental modelling and control was so infectious and who made me realize that this would be one focus of my own future research.

During our work on the modelling of dissolved oxygen and biochemical oxygen demand (DO-BOD) variations in the River Cam, Bruce and I realized that the conventional ‘hypothetic-deductive’ approach to modelling such systems was limited by the difficulty of performing planned experiments. It became clear that more research was required on an alternative ‘inductive’ approach, where the model structure was not assumed *a priori*, but was inferred statistically from real experimental

or monitored data. And it was such thinking that shaped both of our subsequent careers and led, in my case, to the development of the DBM approach to modelling; an approach that has been such a strong motivation for the development of the recursive methods considered in the present book and which is reviewed in its final chapter 12.

Of course, there are many others outside of Lancaster, the ANU and Cambridge who have influenced my work over many years. Just to mention those with whom I have worked and whose friendship I particularly value: John Norton, who I have known for many years and is the author of a well known book on system identification; Howard Wheater, a distinguished hydrologist and long-time friend from the 1970s who, together with Neil Macintyre, Thorsten Wagener and others, has promoted the use of DBM modelling in hydrological applications (see chapters 7, 8, 10 and the Epilogue); Antonio Garcia Ferrer who has strongly supported the application of CAPTAIN tools for unobserved component modelling to economic forecasting (see chapter 5); Keith Burnham who has always been supportive of my research; Barry Croke and Ian Littlewood, with whom I have worked recently (see chapter 8); Marco Ratto and Andrea Pagano, who have been my main collaborators on the development of state dependent parameter regression and large model emulation methods (see chapters 11 and 12); Hugues Garnier and Marion Gilson, who have helped me develop further the RIVC approach to the identification and estimation of continuous-time transfer function models (see chapter 8) and made other important contributions in this area; and Liuping Wang, with whom I have worked on model-based predictive control and state-dependent parameter modelling.

Liuping is a very generous and gifted academic who, with the assistance of Hugues Garnier and Tony Jakeman, was kind enough to put an enormous amount of time into two recent activities that have left me with extremely pleasant memories and provided the stimulus for me to write the present book. First, the organisation of a Workshop in Melbourne, Australia, that marked my 70th birthday in December 2009; and second, the preparation of a book *System Identification, Environmetric Modelling and Control System Design* which collects together 26 contributions to the Workshop and will be published this year. My grateful thanks also to the many other friends and colleagues from all over the World who were kind enough either to attend the Workshop or contribute chapters to the book.

Of course, writing a book is one thing, checking it is quite another. I am immensely grateful to my young friends and excellent research workers, Tomasz Larkowski and Ivan Zajic, who have laboriously checked the draft, noted my many careless errors and made useful suggestions that have considerably improved the book. Of course, I remain solely responsible for any remaining errors.

Finally and most importantly, I wish to thank my wife Wendy, who has been my partner for over fifty years. Without her selfless support and encouragement, I could not have pursued my studies or have written this book.

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